MPC Feed-Forward for constraint handling

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Photo: Jonas Westling

Agenda

- Idea

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- Theory

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- Theory
- Toy example

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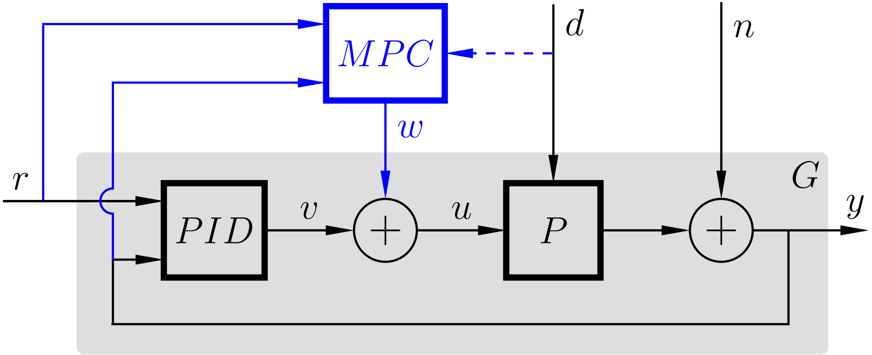
- Idea
- Theory
- Toy example
- Real example



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$$P : \begin{cases} \dot{\boldsymbol{x}}_p = A_p \boldsymbol{x}_p + B_p u & \text{(1a)} \\ y = C_p \boldsymbol{x}_p + D_p u & \text{(1b)} \end{cases}$$

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$$

$$
C : \begin{cases} \dot{\boldsymbol{x}}_c = A_c \boldsymbol{x}_c + B_{cr} r + B_{cy} y & \text{(2a)} \\ v = C_c \boldsymbol{x}_c + D_{cr} r + D_{cy} y & \text{(2b)} \end{cases}
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$$u = v + w \tag{3}$$

$$\dot{\boldsymbol{x}}_p = A_p \boldsymbol{x}_p + B_p (v + w) \tag{4a}$$

$$\dot{\boldsymbol{x}}_c = A_c \boldsymbol{x}_c + B_{cr} r + B_{cy} y \tag{4b}$$

$$v = C_c \boldsymbol{x}_c + D_{cr} r + D_{cy} y \tag{4c}$$

$$y = C_p \boldsymbol{x}_p + D_p (v + w) \tag{4d}$$

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$$y - D_p D_{cy} y = C_p \boldsymbol{x}_p + D_p C_c \boldsymbol{x}_c + D_p D_{cr} r + D_p w \tag{5}$$

$$y - D_p D_{cy} y = C_p \boldsymbol{x}_p + D_p C_c \boldsymbol{x}_c + D_p D_{cr} r + D_p w \tag{5}$$

$$E_c = (I - D_p D_{cy})^{-1} \tag{6}$$

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$$E_c = (I - D_p D_{cy})^{-1} \tag{6}$$

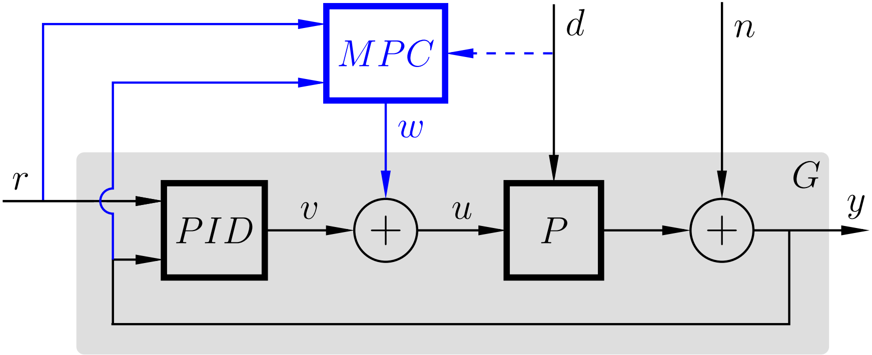
$$y = E_c C_p \boldsymbol{x}_p + E_c D_p C_c \boldsymbol{x}_c + E_c D_p D_{cr} r + E_c D_p w \tag{7}$$

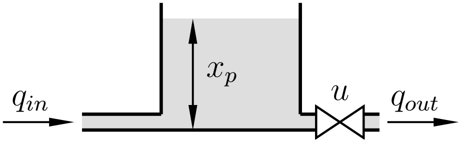
$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_p \\ \boldsymbol{x}_c \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_p \\ \boldsymbol{x}_c \end{bmatrix}$$

$$\dot{\boldsymbol{x}} = \underbrace{\begin{bmatrix} A_p + B_p E_p D_{cy} C_p & B_p E_p C_c \\ B_{cy} E_c C_p & A_c + B_{cy} E_c D_p C_c \end{bmatrix}}_{A} \boldsymbol{x}$$

$$+ \underbrace{\begin{bmatrix} B_p E_p D_{cr} \\ B_{cr} + B_{cy} E_c D_p D_{cr} \end{bmatrix}}_{B_r} r + \underbrace{\begin{bmatrix} B_p + B_p E_p D_{cy} D_p \\ B_{cy} E_c D_p \end{bmatrix}}_{B_w} w$$


MPC
d
n
w
G
r
y
PID
v
u
P



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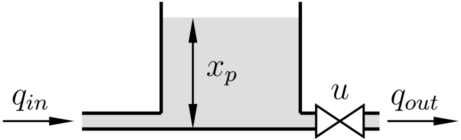

$$
\begin{aligned}
\underset{w}{\text{minimize}} \quad & J(\alpha), \\
\text{subject to} \quad & G, \\
& 450 \leq x_p.
\end{aligned}
$$

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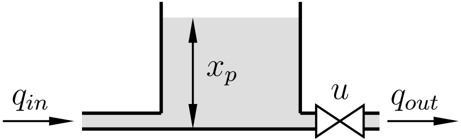

$$
\begin{aligned}
\underset{w}{\text{minimize}} \quad & J(\alpha), \\
\text{subject to} \quad & G, \\
& 450 \leq x_p.
\end{aligned}
$$

$$J(\alpha) = \int_0^h (1-\alpha)\left(r(t) - y_f(t)\right)^2 + \alpha w^2(t)\, dt.$$

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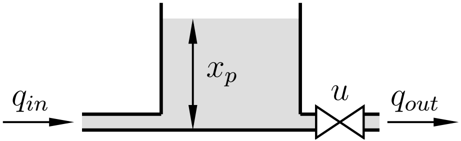
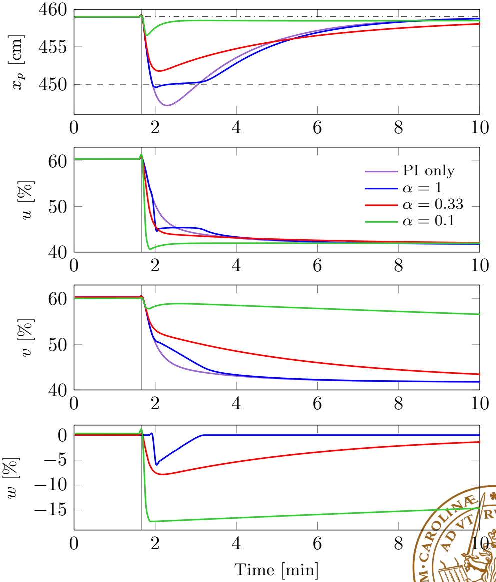


$$
\begin{aligned}
\underset{w}{\text{minimize}} \quad & J(\alpha), \\
\text{subject to} \quad & G, \\
& 450 \leq x_p.
\end{aligned}
$$

$$J(\alpha) = \int_0^h (1-\alpha)\,(r(t) - y_f(t))^2 + \alpha w^2(t)\; dt.$$



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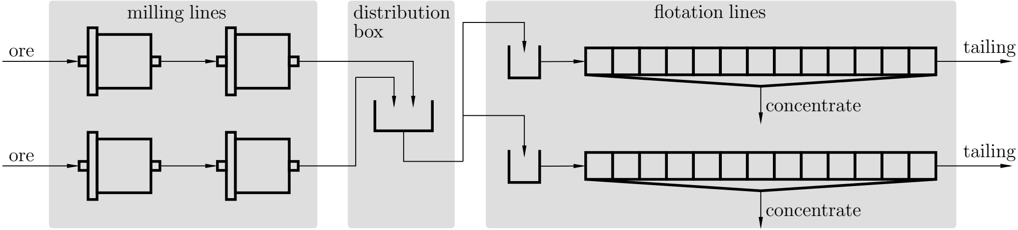
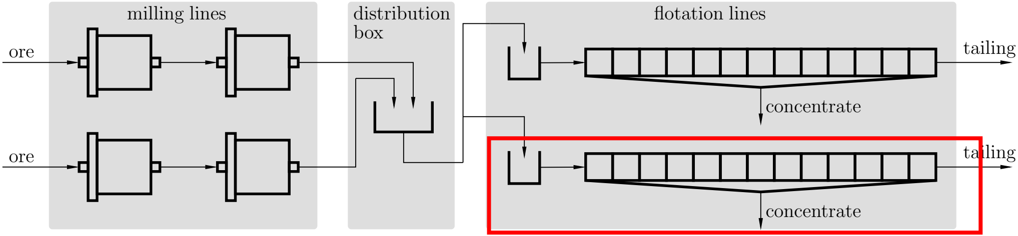

milling lines
distribution box
flotation lines
ore
ore
tailing
tailing
concentrate
concentrate


milling lines
distribution box
flotation lines
ore
ore
tailing
tailing
concentrate
concentrate

$$
\begin{aligned}
\underset{w}{\text{minimize}} \quad & J = \sum_{0}^{h} \boldsymbol{x}^T Q \boldsymbol{x} + \boldsymbol{w}^T R \boldsymbol{w} + \Delta \boldsymbol{w}^T S \Delta \boldsymbol{w}, \\
\text{subject to} \quad & G, \\
& constraints.
\end{aligned}
$$

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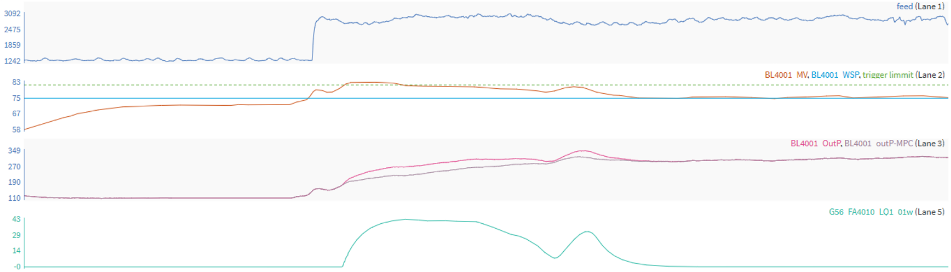
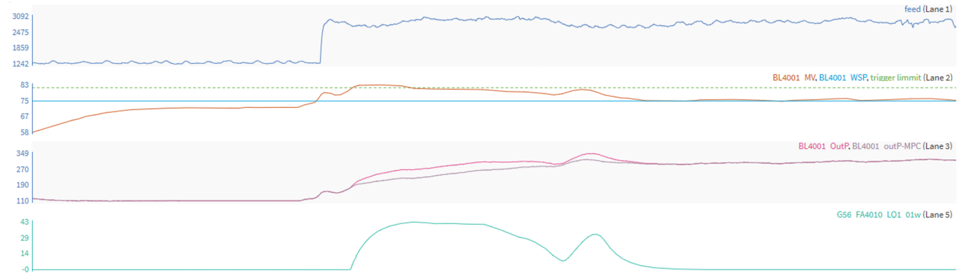

feed (Lane 1)
3092
2475
1859
1242
BL4001 MV, BL4001 WSP, trigger limmit (Lane 2)
83
75
67
58
BL4001 OutP, BL4001 outP-MPC (Lane 3)
349
270
190
110
G56 FA4010 LQ1 01w (Lane 5)
43
29
14
-0


feed (Lane 1)
3092
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BL4001 MV, BL4001 WSP, trigger limmit (Lane 2)
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BL4001 OutP, BL4001 outP-MPC (Lane 3)
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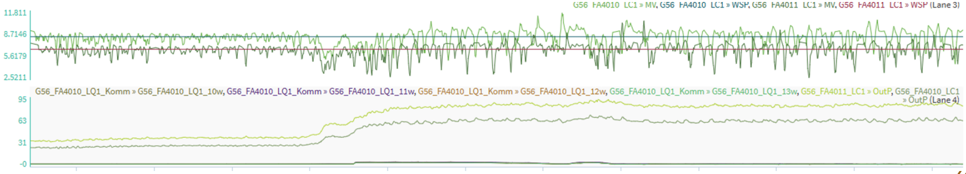
G56 FA4010 LC1 » MV, G56 FA4010 LC1 » WSP, G56 FA4011 LC1 » MV, G56 FA4011 LC1 » WSP (Lane 3)
11.811
8.7146
5.6179
2.5211
G56_FA4010_LQ1_Komm » G56_FA4010_LQ1_10w, G56_FA4010_LQ1_Komm » G56_FA4010_LQ1_11w, G56_FA4010_LQ1_Komm » G56_FA4010_LQ1_12w, G56_FA4010_LQ1_Komm » G56_FA4010_LQ1_13w, G56_FA4011_LC1 » OutP, G56_FA4010_LC1 » OutP (Lane 4)
95
63
31
-0


Thank you
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