

Learningbased HealthAware Control for Improved Process Safety

A Case Study of ~~Gals~~Wells

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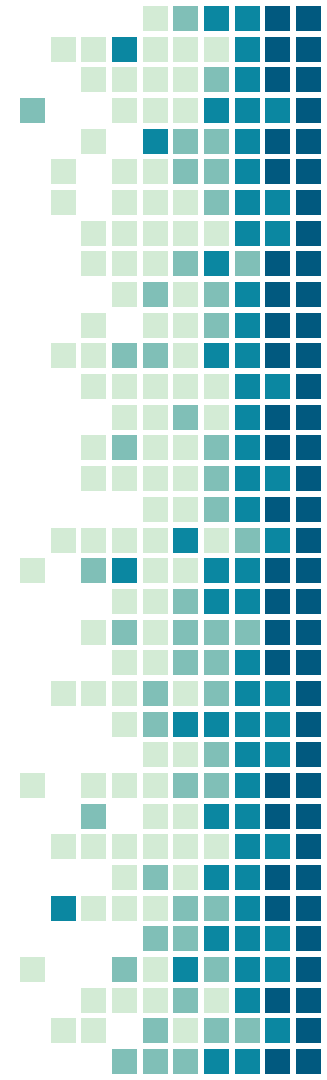
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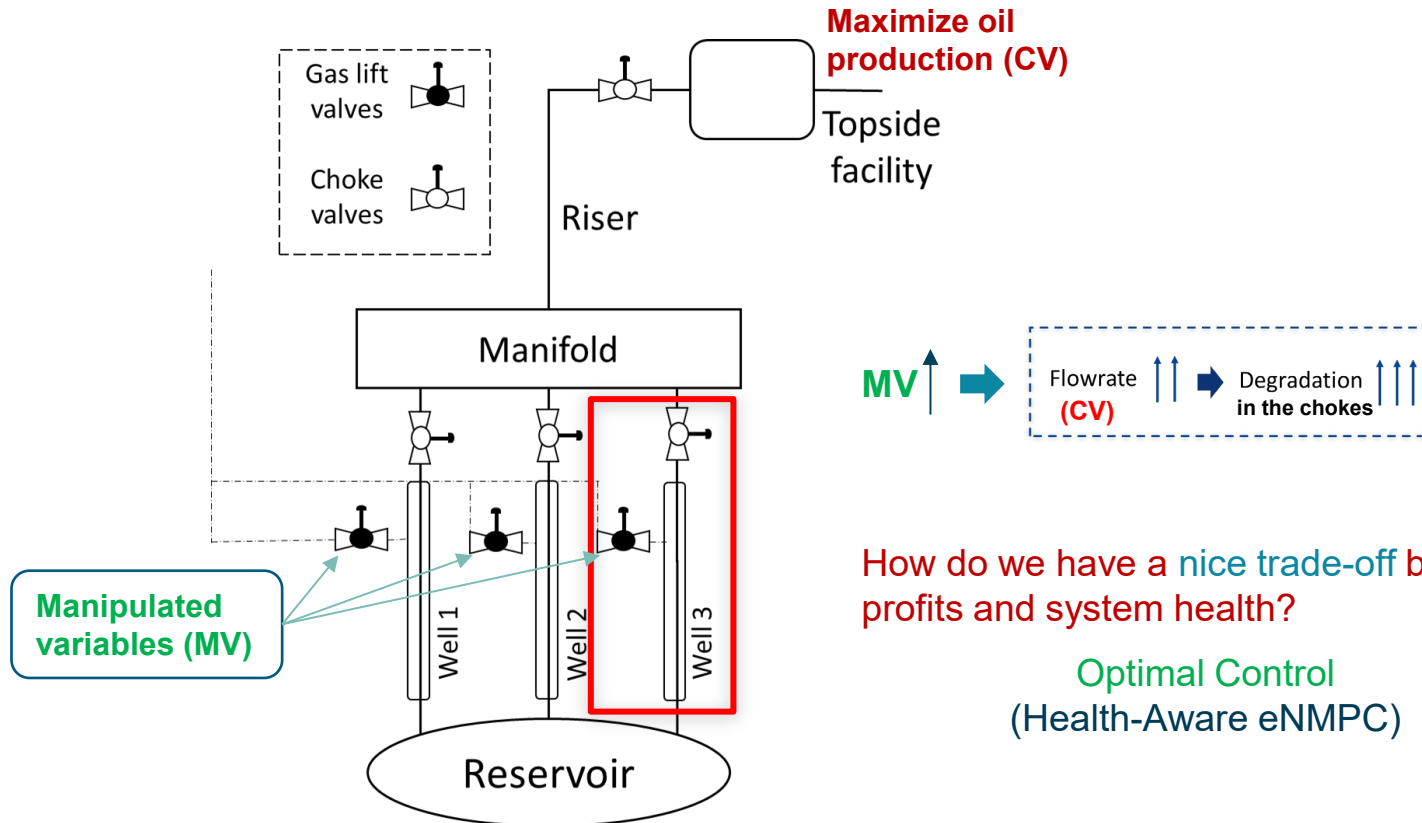
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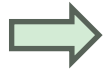
Case Study: Gas Well Networks

Motivation Profit Health TradeOff



HAC Formulation

Maximize oil production
in a planned time
subject to process and
health constraints



$$V_N(\hat{x}) = \max_{u, x} \sum_{k=t_0}^{N-1} w_{po,k} - 0.5 \Delta u_k^T R \Delta u_k - \rho S_k$$

Subject to :

$$[x_{t_0}, u_{t_0-1}] = [\hat{x}, \hat{u}]$$

$$x_{k+1} = F_1(x_k, u_k)$$

$$0 = F_2(z_k, u_k)$$

$$y_k = C(x_k, u_k)$$

$$G(x_k, u_k) - s_k \leq 0$$

$$s_k \geq 0$$

$$u_{min} \leq u_k \leq u_{max}$$

$$\Delta u_k = u_k - u_{k-1}$$

$$|\Delta u_k| \leq \Delta u_{max}$$

$$\Delta u_k = 0$$

$$k \in [t_0, t_0 + N - 1]$$

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$$k \in [t_0, t_0 + N_c - 1]$$

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$$k \in [t_0 + N_c, t_0 + N - 1]$$

Models (DAEs)

Gas-lift model (steady state)

Phase balance model

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{aligned} &\dot{m}_{ga} = \dot{m}_{gl} - \dot{m}_{iv}, \\ &\dot{m}_{gt} = \dot{m}_{iv} - \dot{m}_{pg} + \dot{m}_{rg}, \\ &\dot{m}_{ot} = \dot{m}_{ro} - \dot{m}_{po}, \end{aligned}$$

Density model

$$\begin{aligned} \rho_a &= \frac{M_w p_a}{T_a R}, \\ \rho_w &= \frac{\dot{m}_{gt} + \dot{m}_{ot} - \rho_o L_r A_r}{L_w A_w}, \end{aligned}$$

Flow model

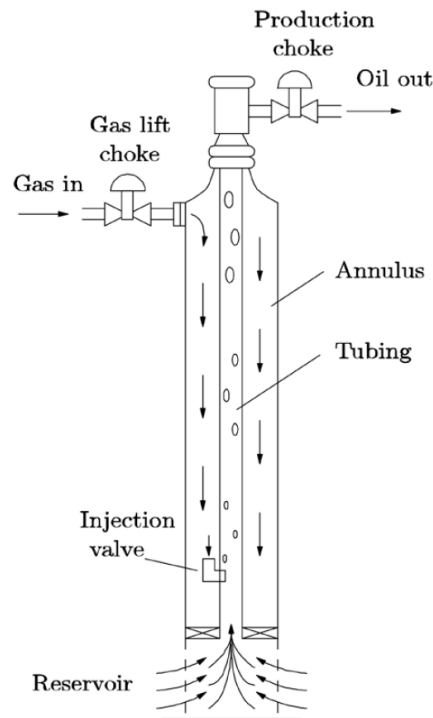
$$\begin{aligned} w_{iv} &= C_{iv} \sqrt{\rho_a \max(0, p_{ai} - p_{wi})}, \\ w_{pc} &= C_{pc} \sqrt{\rho_w \max(0, p_{wh} - p_m)}, \\ w_{pg} &= \frac{\dot{m}_{gt}}{\dot{m}_{gt} + \dot{m}_{ot}} w_{pc}, \\ w_{po} &= \frac{\dot{m}_{ot}}{\dot{m}_{gt} + \dot{m}_{ot}} w_{pc}, \\ w_{ro} &= PI(p_r - p_{bh}), \\ w_{rg} &= GOR \cdot w_{ro}, \end{aligned}$$

Pressure model

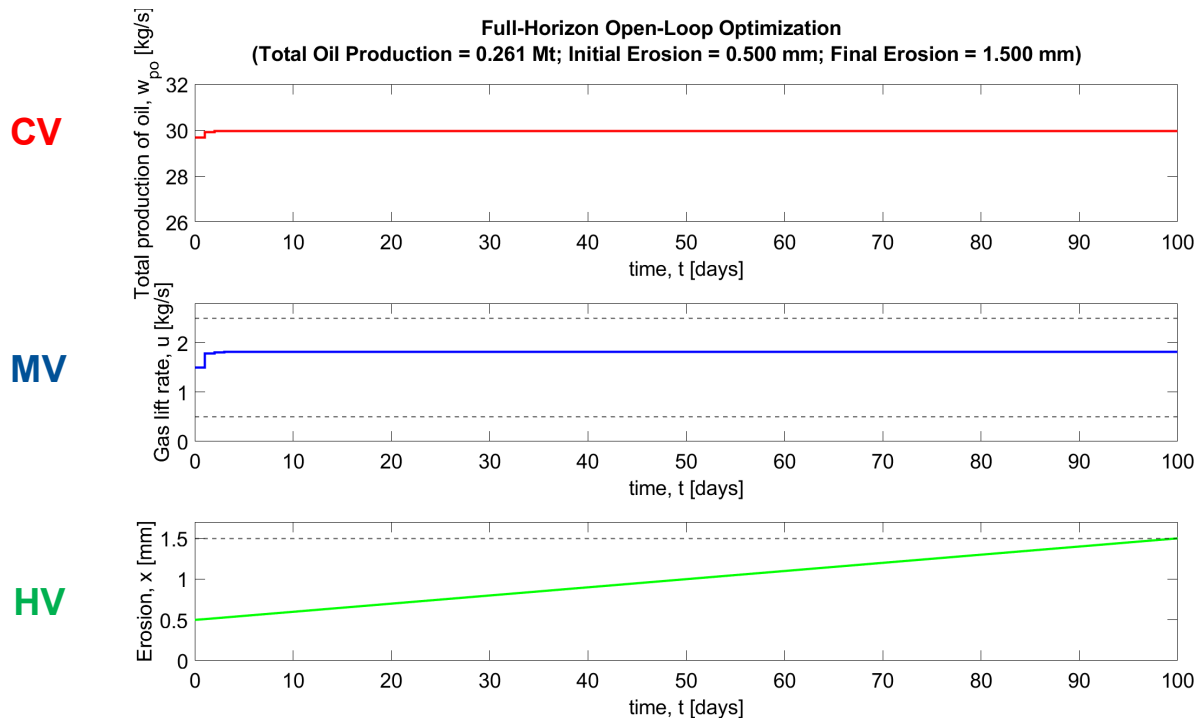
$$\begin{aligned} p_a &= \left(\frac{T_a R}{V_a M_w} + \frac{g L_a}{L_a A_a} \right) \dot{m}_{ga}, \\ p_{wh} &= \frac{T_w R}{M_w} \left(\frac{\dot{m}_{gt}}{L_w A_w + L_r A_r - \frac{\dot{m}_{ot}}{\rho_o}} \right), \\ p_{wi} &= p_{wh} + \frac{g}{A_w L_w} (\dot{m}_{ot} + \dot{m}_{gt} - \rho_o L_r A_r) H_w, \\ p_{bh} &= p_{wi} + \rho_w g H_r, \end{aligned}$$

Erosion model

$$\dot{\epsilon} = \frac{K \cdot F(\alpha) \cdot U_p^n}{\rho_t \cdot A_t} \cdot G \cdot C_1 \cdot G_f \cdot \dot{m}_p \cdot C_{unit}$$



OpenLoop Optimization



We want to achieve the same (or similar) performance FASTER (i.e. lower solve time) using ML models



Fast HACADPflavoured HealthAwareNMPC

Fast HAC Formulation

Basic Idea

Optimize with a **very short horizon** while taking into account the long-term effects in the cost-to-go (CTG)— motivated by **Bellman's principle of optimality**, i.e.:

Needs to be known beforehand

Do

$$\hat{V}_N(\hat{x}) = \min_{u_0} l_0(\hat{x}, u_0) + \hat{V}_{N-1, \mu}(x_1), \quad x_1 = f(\hat{x}, u_0)$$

Instead of

$$V_N(\hat{x}) = \min_{u_0, u_1, \dots, u_{N-1}} \sum_{k=0}^{N-1} l_k(x_k, u_k)$$

Ways to learn the CTG term (optimal value function):

1. (Approximate) Dynamic Programming CTG term approximated iteratively
2. Learning-based MPC: supervised learning of CTG labels

Algorithm for Creation of ~~One~~Step eNMPC

1. Generate labels:

- Perform **full-horizon open-loop optimization** to obtain the optimal CTG values $\hat{V}_N(\hat{x})$ for sampled *initial erosion values* $(\hat{x})$ and *prediction horizon* (N) .

2. Train a surrogate (approximate) model, $\hat{V}_N(\hat{x})$:

- Fit an **input-convex neural network (ICNN)** model to the training data

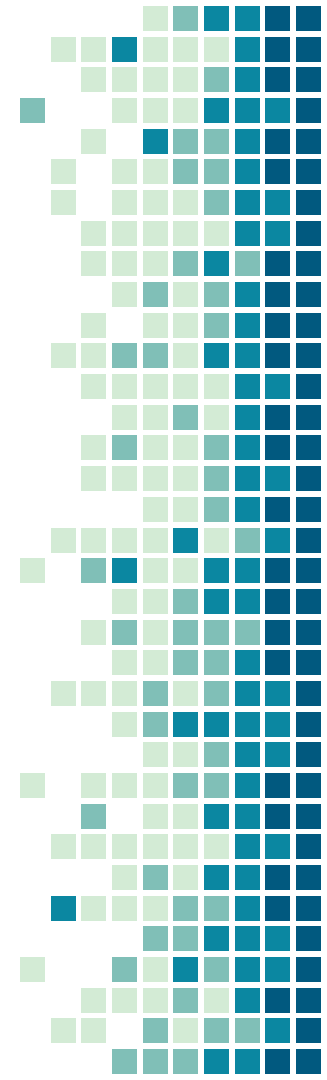
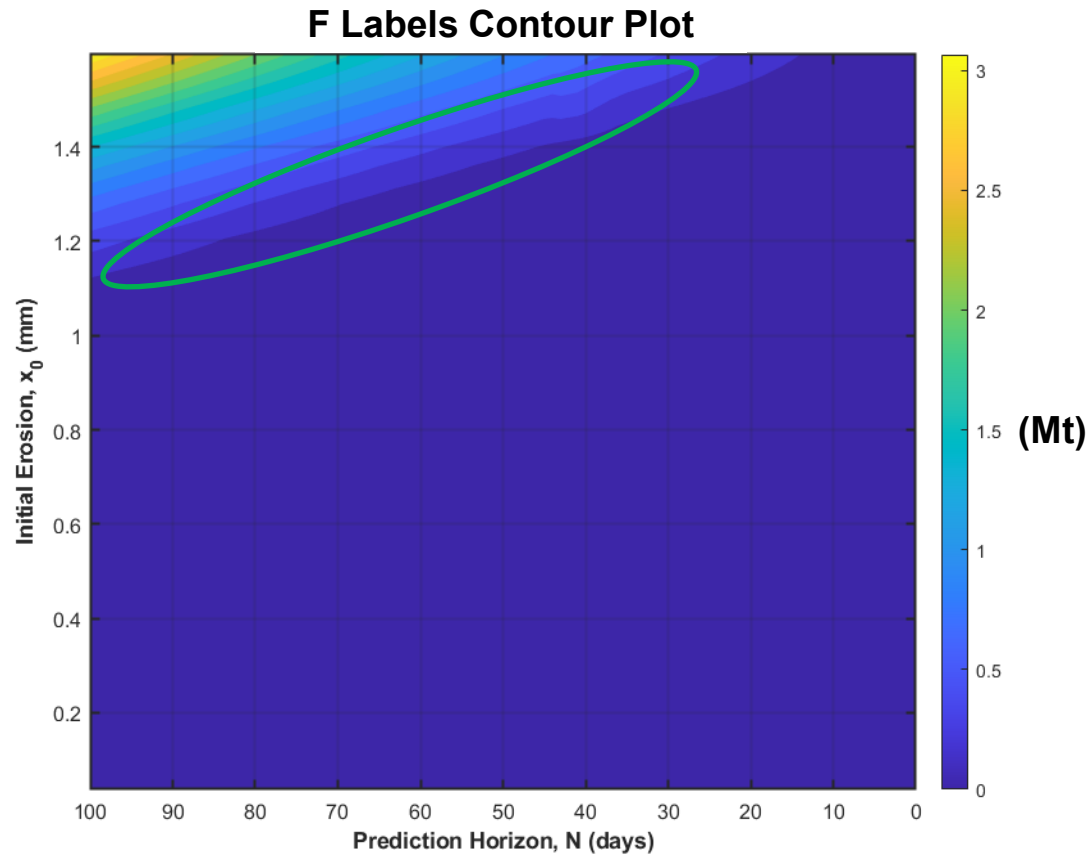
3. Deploy one-step eNMPC

- Solve the one eNMPC with $\hat{V}_{N-1,\mu}(x_1)$ as the terminal cost.

NB: To better handle future constraints (feasibility), the CTG surrogate is split into two, i.e.:

$$\hat{V}_{N-1,\mu}(x_1) = \underbrace{\hat{V}_{N-1}(x_1)}_{\text{Optimality}} + \underbrace{\mu \hat{F}_{N-1}(x_1)}_{\text{Feasibility}}$$

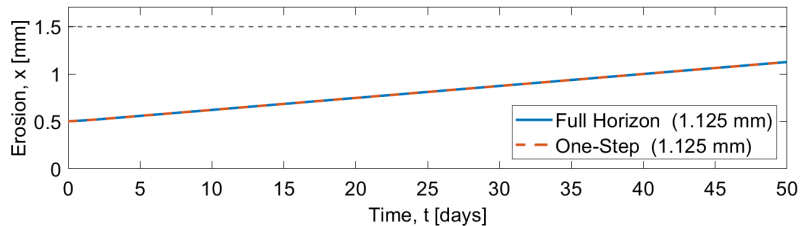
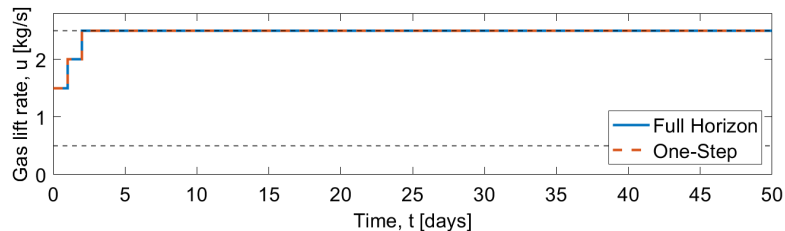
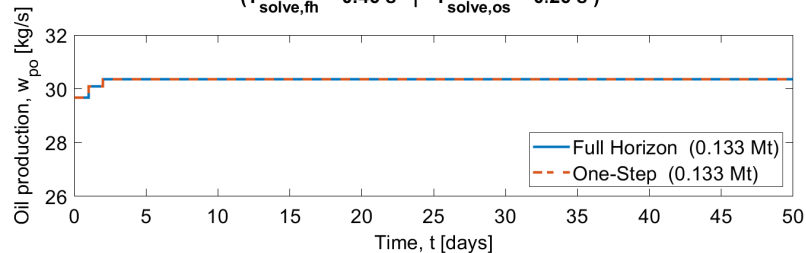
Feasibility Data Visualization



ClosedLoop Implementation*

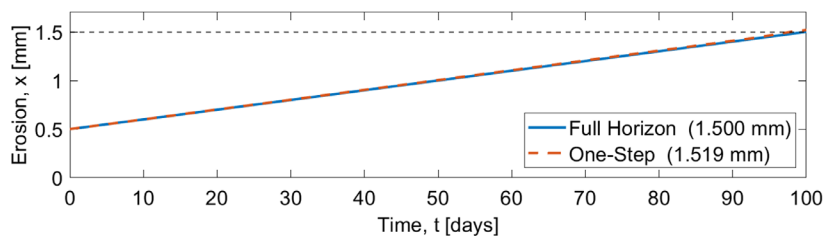
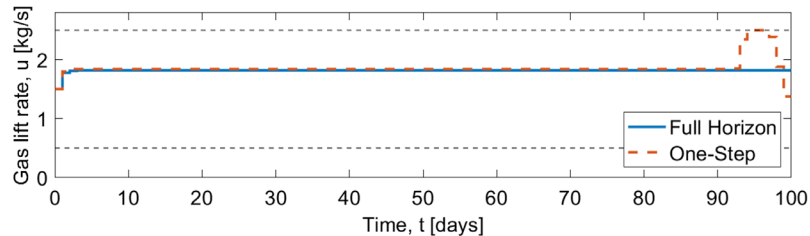
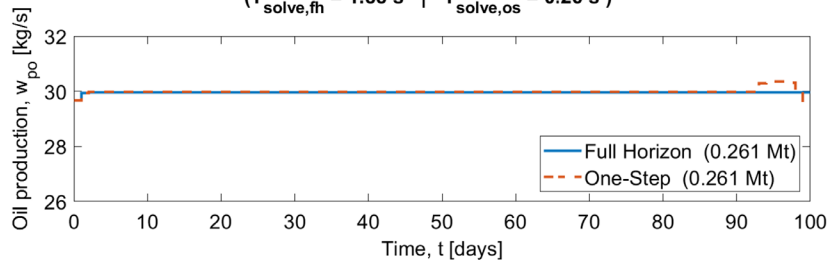
Feasible Region

Full-Horizon eNMPC vs One-Step eNMPC
($T_{\text{solve, fh}} = 0.46 \text{ s}$ | $T_{\text{solve, os}} = 0.26 \text{ s}$)



Boundary Region

Full-Horizon eNMPC vs One-Step eNMPC
($T_{\text{solve, fh}} = 1.88 \text{ s}$ | $T_{\text{solve, os}} = 0.26 \text{ s}$)



*no plant-model mismatch

Remarks on Oil Well Case Study

- The one-step controller behaves exactly (or close) to the full-horizon controller in the **feasible region**:
 - Need better **F surrogate** performance at the boundary of infeasibility (around & at kink).

Ideas	Action	Done or not?	Improvement?
Focused training for F surrogate – better local prediction	Weighted MSE	✓	No
Learn 1st derivative information better to capture the 'kink' Add a regularization term on gradients during training	Gradient targets from data	✓	No
	Using dx as a 3rd feature	✓	No
	Using costates as gradient targets since $(\lambda^*(t) = \nabla_x V(x^*(t), t))$	In progress	-
Alternative models and formulations	Reformulate as MILP: a ReLU ANN	✗	-
	Use RNNs as a surrogate	✗	-

Conclusions & Outlook

- A learned ICNN cost-to-go can reproduce long-horizon NMPC behavior in the feasible operating region.
- Near active health constraints, performance depends strongly on the (feasibility) surrogate's accuracy and sensitivity around the feasible set boundary.
- Future work will focus on constraint-aware training, sensitivity matching, and extension to MIMO uncertain systems.

Thank you for your attention!

