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Dynamic Bioreactor with Automated Back-Off Calculating Nonlinear MPC for Increased Robustness

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Methodology

Process models rely on parameters from literature, but real values often differ or are uncertain — causing model-mismatch. ABC-NMPC (Krog et al. [1]) addresses this by using fast Monte Carlo (MC) simulations to compute a constraint "back-off" that accounts for this uncertainty in real time.

NMPC Structure:

- Prediction/control horizon n_p / n_c = 5 hr/3 hr
- Soft constraints, no back-off
- Uncertain parameters are resampled from a normal distribution after every time step

ABC-NMPC Structure:

- Same structure as NMPC
- Back-off recalculated at every step via one-step-ahead MC prediction on normal distributed state parameter values which act as disturbances d_i
- The MC-predictions calculate one time step ahead from t_k to t_{k+1}
- Back-off = worst-case constraint violation predicted over MC samples

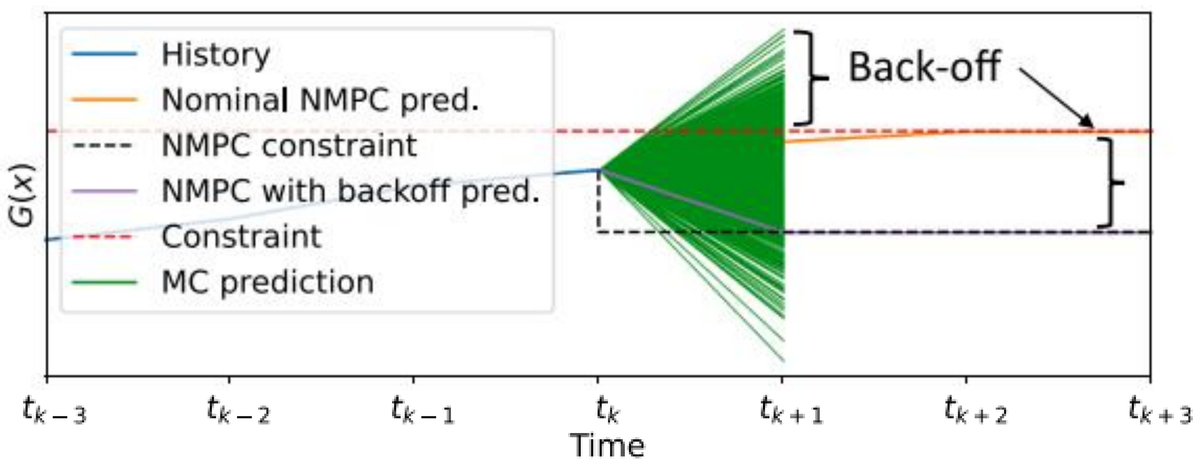

Fig. 1 Visualization of ABC-NMPC algorithm. The highest vionlat of constraints in the MC-prediction for the next time step is chosen as back-off b_j [1], edited.

Case Study

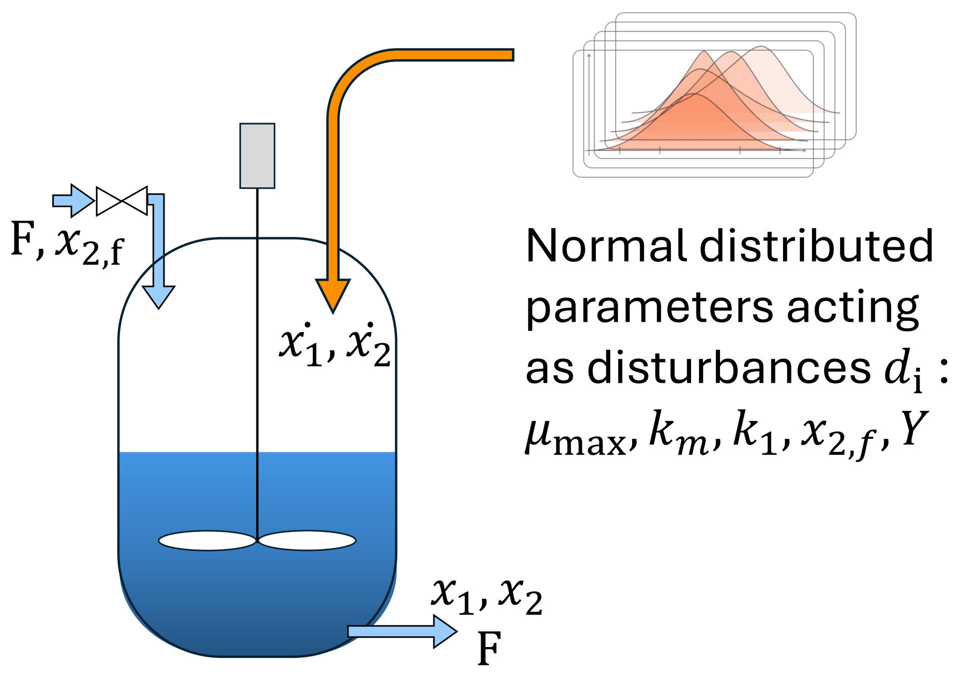

Fig. 2 CSTR-bioreactor. The only input is the inflow F which is implemented over the dilution rate D. The disturbances are given over the normal distribution of the parameters.

Model:

States (CV):
- x_1 = Biomass concentration
- x_2 = Substrate concentration

Variables:
- μ_{si} = Specific growth rate
- F = Volumetric flow rate

Inputs (MV):
- D = Dilution rate (F/V)

Constraint violation metric:
- $\int_0^{60} max(0, x_{2,min} - x_2(t))dt$

State Equations:

$$\dot{x_2} = D(x_{2,f} - x_2) - \frac{\mu_{si} x_1}{Y}$$

$$\dot{x_1} = (\mu - D)x_1$$

Substrate Inhibition:

$$\mu_{si} = \frac{\mu_{max} x_2}{k_m + x_2 + k_1 x_2^2}$$

NLP:

$$\min_{x,u,\epsilon} \frac{1}{2}\left(\sum_{j=1}^{n_p} \left(y_{1,j} - y_{SP,1,j}\right)^T Q \left(y_{1,j} - y_{SP,1,j}\right) + \sum_{j=1}^{n_m} \Delta u_j^T R \, \Delta u_j\right) + \rho \sum_{j=1}^{n_p} \epsilon_j$$

s.t. Standard OCP formulation

$$x_{2,j} \geq x_{2,min} + b_j \qquad j = 1, \ldots, n_c$$

Nominal Case

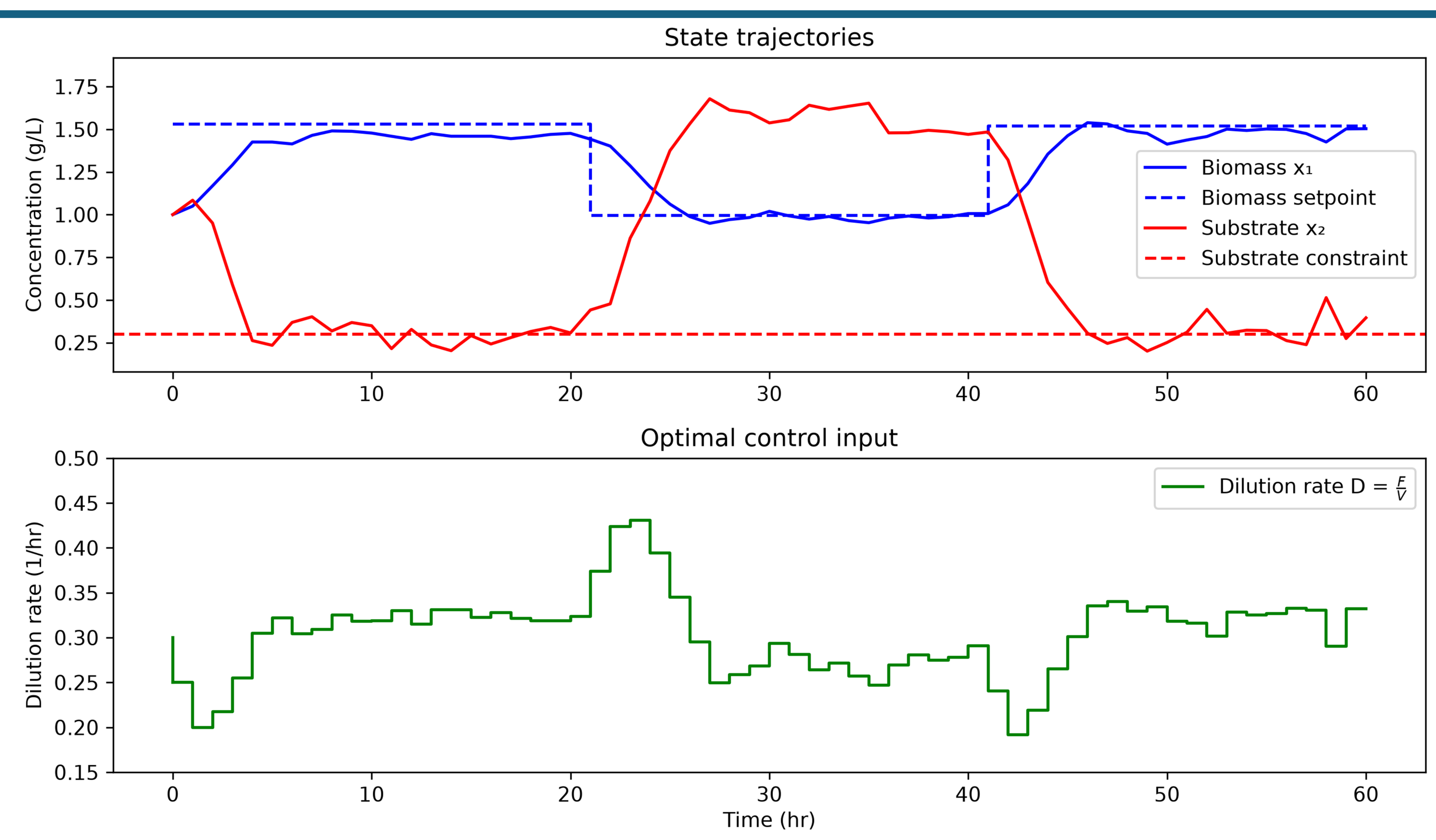

Fig. 3 Result of the NMPC. Standard deviation of σ = 5% of the parameters around their mean was chosen as disturbance. The minimum substrate constraint is violated several times.

Robust Control Structure

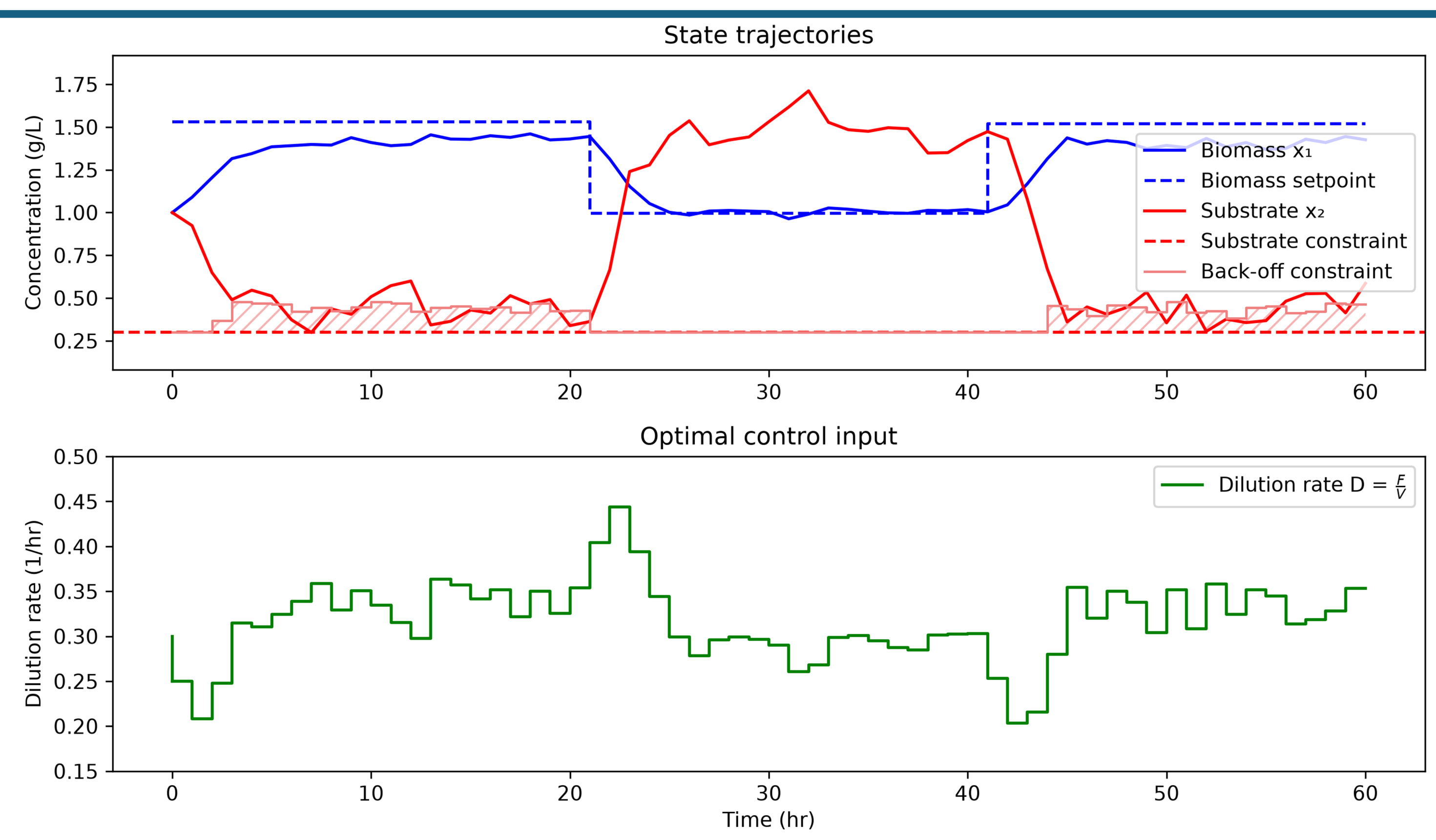

Fig. 4 Result of the ABC-NMPC. Standard deviation of σ = 5% of the parameters around their mean was chosen as disturbance. Significant improvement in robustness is seen.

Tab. 1 Mean (median) of the constraint violations for the ABC and nominal NMPC as well as the solve time for the corresponding NMPC problem.

MC-Samples	σ	ABC constraint viol.	ABC solve time [s]	Nominal constraint viol.	Nominal solve time [s]
100	5	0.07 (0.05)	1.21 (1.16)	0.86 (0.87)	0.57 (0.55)
100	10	**0.34 (0.30)**	1.15 (1.14)	1.51 (1.49)	0.54 (0.53)
1,000	5	0.06 (0.04)	3.12 (3.03)	0.85 (0.83)	0.56 (0.55)
1,000	10	0.36 (0.36)	3.46 (3.34)	1.51 (1.50)	0.59 (0.57)
10,000	5	**0.03 (0.02)**	43.16 (26.48)	0.87 (0.87)	0.93 (0.63)
10,000	10	0.35 (0.30)	31.20 (23.92)	1.52 (1.50)	0.65 (0.51)

Conclusion

- On average the ABC-NMPC reduces the constraint violation metric **93.9%** (**76.8%**) for 5% (10%) standard deviation.
- 5% standard deviation case shows clear decrease in constraint violations with growing MC-samples whereas for the higher 10% case lowest result is observed for lowest number of MC-samples.
- The best result is achieved for 5% standard deviation and 100 MC-samples.
- Solver times increase significantly with increasing MC-samples compared to nominal base case. Lowest number of MC-samples shows the best return on computation.

Sources

[1] H.A. Krog, J. Jäschke, "A simple and fast robust nonlinear model predictive control heuristic using n-steps-ahead uncertainty predictions for back-off calculations", JoPC, vol. 141, Year 2024
[2] B.W. Bequette, "Process dynamics: modeling, analysis, and simulation", Pearson, Module 8, Year 2025.

Acknowledgments

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